The History of Integral Print Methods

An excerpt from:
"Lens Array Print Techniques"

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Integral imaging is a true *auto-stereo* method (stereo imagery viewable without the requirement of special glasses). An integral image consists of a tremendous number closely packed distinct micro-images, that are viewed by an observer through an array of spherical convex lenses, one lens for every micro-image. This special type of lens array is known as a *fly's-eye* or *integral* lens array; Fig. 1.

When properly practiced, the result is stunning three dimensional imagery that coveys a realism matched only by museum-quality holograms. Indeed, it has been demonstrated that an integral image can very accurately reproduce the wavefront that emanated from the original photographed or computer generated subject, much like a hologram, but without the need for lasers to create the image. This allows the eyes to accommodate (focus) on foreground and background elements, something not possible with lenticular or barrier strip methods. The term "Integral" comes from the *integration* of all the micro images into a complete three dimensional image through the lens array. In addition to three dimensional effects, elaborate animation effects can also be achieved in integral images, or even a combination of these effects.

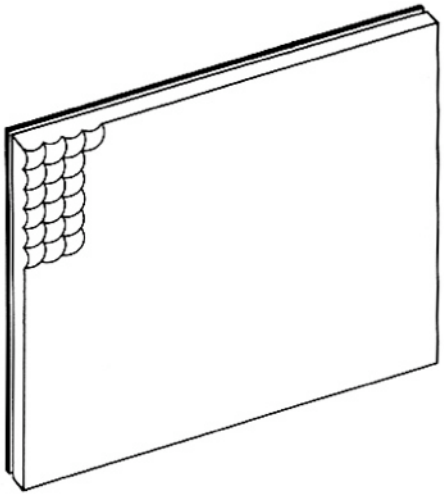

Figure 1: Fly's eye lens sheet illustration.

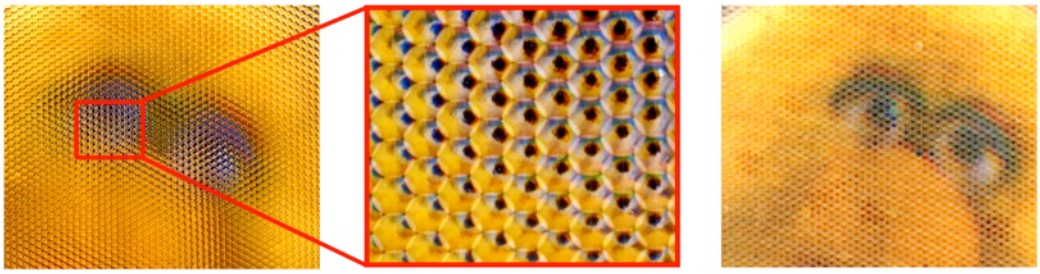

Figure 2: Integral image (Left) without lens; Enlargement (Center), note that each lens records its own unique picture; Integral image (Right) resolved through a matching lens from a particular viewing position; Roberts & Villums, 1989.

Integral imaging is based on a principle known as the lens "sampling effect". To achieve this effect, the thickness of the lens array sheet is chosen so that parallel incoming light rays generally focus on the opposing side of the array, which is typically flat; see Fig. 3 (far right image). This flat side is known as the focal plane. It is at this plane that the micro images are placed, one for every lens, side by side. Since each lenslet focuses to a point onto a micro image below, an observer can never view two spots within a micro image simultaneously; just one spot at a time, depending at what angle the observer looks though the lens. For example, if you have an array of small white dots, on an otherwise black background, behind each lens at the focal plane, any given lens will appear either completely black or white, depending on whether or not the lens is focused on a white dot, or the black background; Fig. 3 (left). The state of each lens will vary depending on the point of observation. If all the dots are precisely ordered in a pre-calculated way, a completely different composite image can be directed to each eye of an observer, simultaneously, since each eye looks through the lens array at a different angle. The resolution of an integral image is therefore directly determined by the density of lenses in the array, since each lens effectively becomes a "dot", or pixel (picture element), in the picture, with the visual state of each dot being a function of the viewing angle.

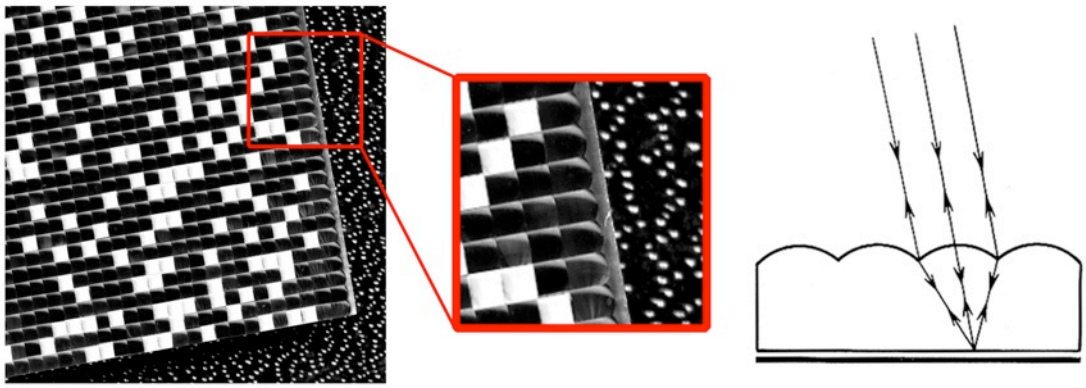

Figure 3: Sampling effect using a fly's eye lens array placed on a printed image of white dots on a black background; Left, Roberts. Sampling effect illustration; Okoshi, Academic Press, 1976 (64).

Unfortunately, in the early days of integral imaging, lens arrays were nearly non-existent. It wasn't until World War II that inexpensive and formable thermal plastics became widely available and methods evolved to form the new materials into arrays. Before then, most of the research utilized arrays of the optically analogous *pin-hole* aperture. In fact, a pin-hole aperture is a lens, but a lens based on diffraction rather than refraction. It is essentially a clear aperture, or hole, in an otherwise opaque plate. In photography, the radius of the pin-hole is selected to focus upon the film a concentrated and well distributed image of an object, where the focal length is approximately the wavelength of light divided into the square of the radius of the pin-hole. To use a pin-hole array as a viewing screen, the radius is typically much larger to allow more light to pass through the array. However, to properly to view an integral image, the aperture must generally be ten percent or less open, which results in a very dark image, even when a bright light is used for back lighting.

The first integral imaging method was "Integral Photography". In this method the lens array is used to both record and play back a composite three-dimensional image. When an integral lens array sheet is brought into contact with a photographic emulsion at its focal plane, and an exposure is made of an illuminated object that is placed close to the lens side of the sheet, each individual lens (or pin-hole) will record its own unique micro image of the object. The content of each micro image changes slightly based on the position, or vantage point, of the lenslet on the array. In other words, the integral method produces a huge number of tiny, juxtaposed pictures behind the lens array onto the film. After development, the film is realigned with the lens sheet and a composite spatial reconstruction of the object is re-created in front of the lens array, that can be viewed from arbitrary directions within a limited viewing angle.

Integral was the first lens-based auto-stereo method, followed by lenticular in the 1930's. It was first proposed on March 3rd, 1908 by physicist Professor Gabriel Lippmann[1] to the French Academy of Sciences, under the title "La Photographie Integral"(56). He proposed a method to record a complete spatial image on a photographic plate, with parallax in all directions, utilizing an array of small spherical convex lenses, all in a single exposure. In his approach, later known as the direct method, an object or scene is recorded directly in front of the lens array. Lippmann proposed this technique without actually ever having proven the concept in experiment. In a second paper in 1908 (55) he described a crude test where he used a screen composed of glass rods with spherical ends where he reported limited success. In a later paper in 1911 (57), he describes a test where he used an array of 12 small lenses mounted in a rectangular frame. He stated that "in illuminating the plate one no longer sees individual microscopic images; they are replaced by a single (integral) image, which is seen under the same angle as the original subject". He went on to report that the resulting image changes form, just like the original object itself, depending on the position of the viewer, and also changes its angular dimensions with distance. He also proposed a 360 degree panorama that could be fixed on a integral cylindrical plate, and even a spherical one that could accommodate all surrounding space.

Figure 4: Professor Gabriel M. Lippmann. A self-portrait using his color photographic process.

The first experiment to verify Lippmann's method was preformed by Professor P..P. Sokolov of the Moscow State University in 1911 (76), using a pin-hole aperture sheet. Although this resulted in a relatively dark image, the experiment was successful in imaging a light bulb filament, that appeared to float off the screen. Sokolov provided a detailed mathematical and experimental description of Lippmann's method and was the first to compute the ideal shape of the back surface of the lens array. He established that integral photographs, "being taken without an objective lens, give, upon direct examination, an impression of relief characteristic of stereoscopic photography, the photographs exhibiting not only a complete relief, but a perspective varying depending on the angle at which one views the plate, that is, an approximation of reality which, until now, has been unattainable in any other instrument."

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[1] Professor Gabriel Lippmann was perhaps best known for his invention of the first photographic reproduction of true color in 1886; Fig. 4. The colors were reproduced by recording standing waves formed within an emulsion layer by the interference of direct and reflected light (the photographic plate was floated in a mercury bath). He was awarded the Nobel Prize for the invention in 1908. The invention was ironically, in essence, the first holographic method.

Estanave of France repeated the experiment in 1930, again creating an image of a bright light filament. He worked with units of 56 and 95 *stanhope* glasses (a type of magnifying glass), and then later used an array of 1250 apertures, which he called *stenopic* cameras (29).

Lippmann's direct method had its limitations. First, it only allowed for objects to be recreated in front of the lens array, in other words, objects appeared to float only in front of the lens array, not within or behind it. Further, because of the limitations in the depth of field of the individual lenslets, the distance an object could be placed in front of the array was limited, and indeed only objects located several centimeters from the array where properly re-imaged.

Herbert E. Ives later improved the technique in 1930, by incorporating a large aperture camera lens (a lens with a diameter wider than the interocular distance between the eyes) to optically suspend a "real" aerial image of an object in front of, within, or behind the lens array. Later known as the indirect method, this allowed for a substantial increase in the depth of field, and for the first time, objects that appeared to float behind the lens array as well as in front. Ives also proposed the use of a large concave mirror as an alternative to the objective lens.

The biggest drawback, however, to the Lippmann method was that the recorded images were pseudoscopic, or depth reversed, where the foreground becomes the background and vice versa. Interestingly, Lippmann himself was apparently not aware of this problem, as he never wrote about it. Herbert E. Ives was the first to recognize the problem in 1931 (50), and proposed a secondary exposure solution to invert the depth. Known as a "two step" method, where a secondary exposure of the original photographic plate through another lens sheet was made. He demonstrated this solution by using a secondary array of pin-hole apertures.

Clarence W. Kanolt also experimented with arrays of pin-hole apertures and large objective camera lenses; Fig. 5. His aperture arrays varied between 40 and 200 per square inch, including hexagonal-packed, square-packed and random arrays. He also proposed using lenses instead of apertures, although it is not clear that any reasonable arrays were available to him at the time.

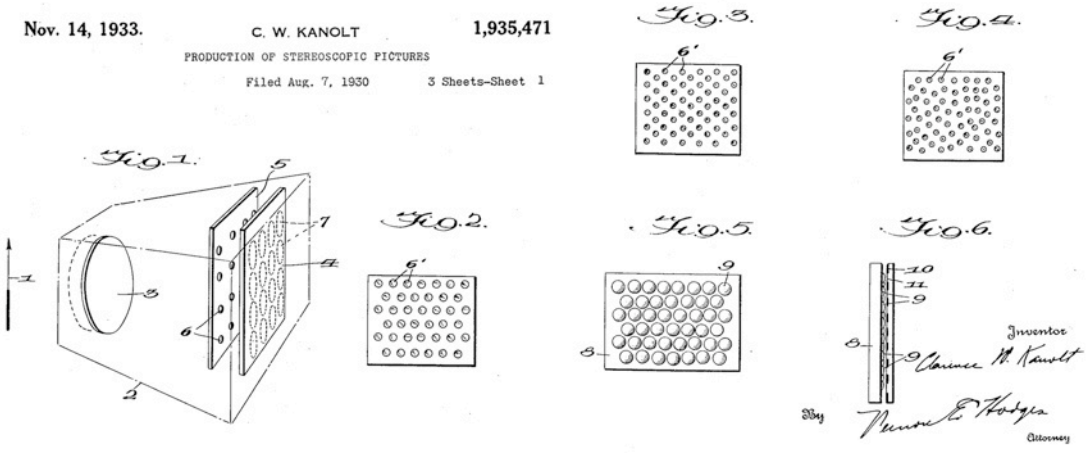

Figure 5: Integral array, U.S. Patent 1,935,471; Kanolt, 1933 (51).

Important refinements were also made to the indirect camera designs and pseudoscopic reversal schemes by the prolific inventor Douglas Winnek of New York in 1936, including establishing a direct relationship between the objective lens design and the design of the lenslets. Winnek constructed elaborate cameras and expanded on the use of objective lens apertures, using a variety of integral screens; Fig. 6.

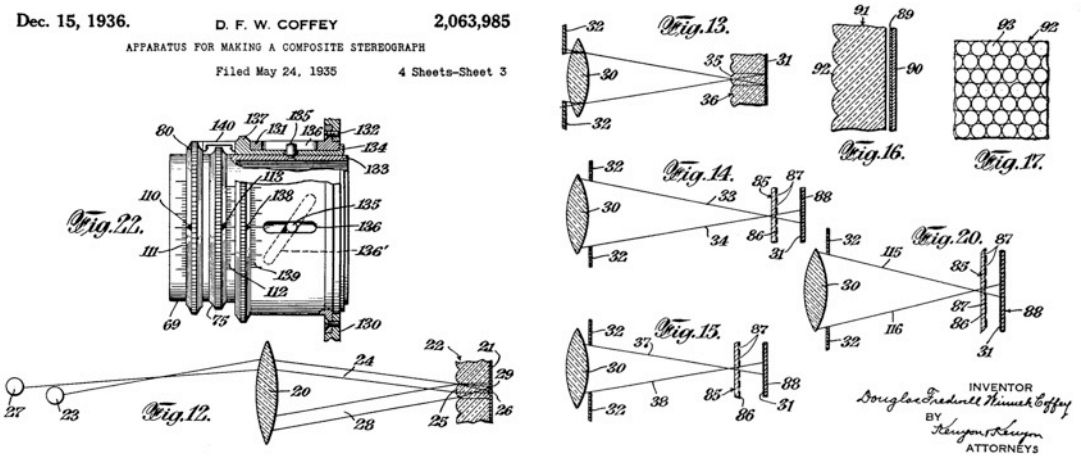

Figure 6: Integral screens and cameras, U.S. Patent 2,063,985; Winnek, 1933 (82).

Investigators Granont and Planovern of France experimented with an integral method that used an array of flat mirrors instead of a lens or pin-hole array (29). The mirrors were precisely placed in such a way that each provided an image of a subject from a slightly different point of view to the film. By projecting the resulting exposure through the same system of a camera and mirrors, a three dimensional image of the original subject was produced.

The first experiments using a proper lens array were apparently performed in 1948 by S.P. Ivanov and L.V. Akimakina of the Soviet Union (80). The lens array reportedly had two million lenses with a diameter of .3 mm (85 lenses per linear inch) and a focal length of .5 mm (.020 inches). This would suggest that the array size was nearly 42 cm (17 inches) square.

Maurice Bonnet of Paris France proposed the first camera method that was capable of recording both dimension and/or motion; Fig. 7. His camera utilized a scanning “selector” mask to record a scene over a short period of time, a method he later mastered using lenticular (cylindrical) arrays. He used either square-packed lens arrays or two, perpendicularly-crossed, lenticular lens arrays.

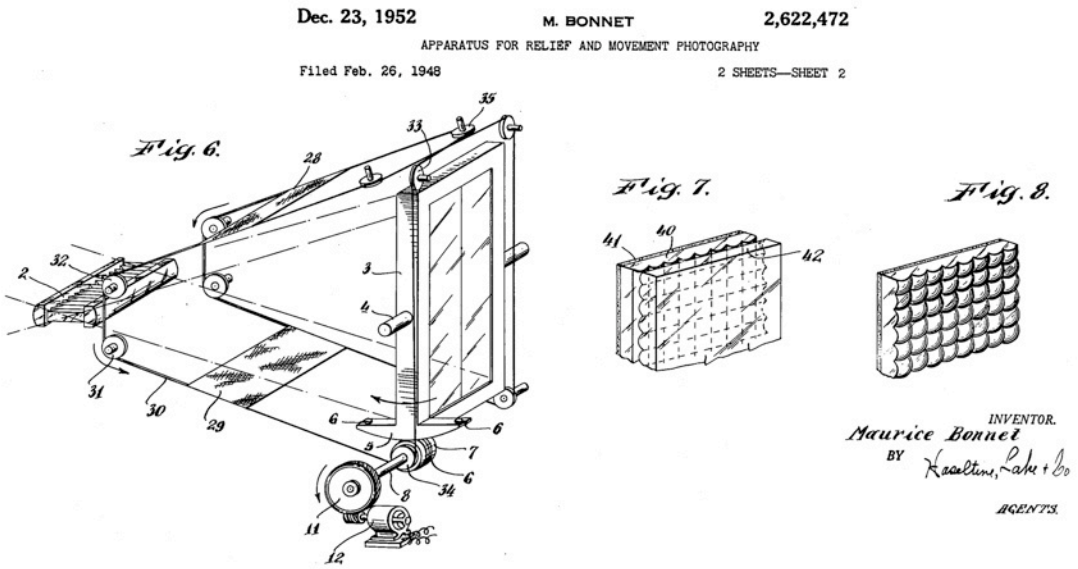

Figure 7: Illustration depicting an "Apparatus For Relief and Movement Photography" using integral lens arrays, U.S. Patent 2,622,472; Bonnet, 1952 (4).

In 1955, John T. Gruetzner of New Jersey experimented with a method of embossing the lens array into plastic photographic film stock, which was subsequently coated with a photographic emulsion; Fig. 8. He produced lens arrays with 40,000 lens per square inch (200 lenses per linear inch). The standard film thickness at the time was .007 inches. His patent included a consumer-level camera design.

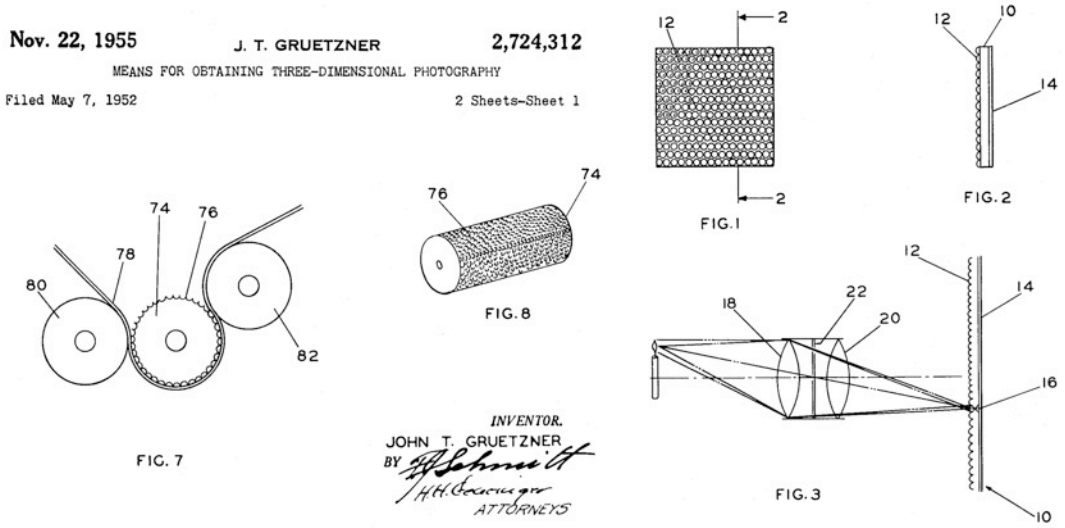

Figure 8: Integral lens array manufacturing and imaging method, US Patent 2,724,312; Gruetzner, 1955 (43).

The general optical principles of the indirect method, using a primary lens, is nearly identical to an ordinary camera, with three exceptions:

First, the objective lens is typically much larger than a normal camera lens; so chosen to accept a wide field of view of an object.

Second, a lens array is placed directly in front of, and often coated with, the light sensitive emulsion, with the lenslet side facing the objective lens.

Third, the image formed by the lens is not brought into focus, instead it is *placed* relative to the lens screen/emulsion layer in such a manner as to recreate the appearance of the object at that position.

As Winnek pointed out in 1936 (82), an important consideration for this camera method is to establish a relationship between the aperture of the primary lens and the field angle of the individual lenslets within the array, to ensure that adjacent sub-images in the image array are precisely abutting, and not appreciably overlapping or spaced apart. In general, the *f*-number of the primary lens must be numerically lower than that of the lenslets. In many instances an adjustable, opaque aperture plate was used to optimize this relationship for different optical arrangements. Lesley Dudley of Los Angeles pointed out that the shape of the aperture can also be adapted to correspond to different lens shapes and packings, such as square aperture for square-packed arrays or a circular aperture for hexagonally-packed arrays (Fig. 9). Both Dudley (25-28) and Takanori Okoshi (61-65) have provided exceptional studies of the optics of a variety of integral methods.

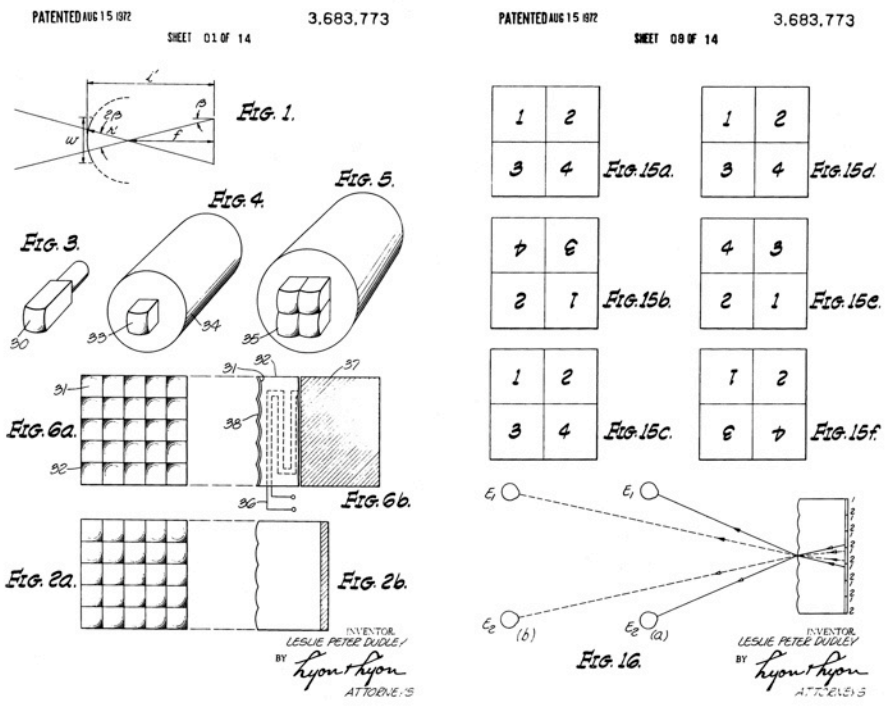

Figure 9: Integral lens array configurations, US Patent 3,683,773; Dudley, 1972 (26).

One key advantage of the indirect method, which often consisted of an compound optical assembly, is that the location of the aerial “real” image could be modified by either adjusting the location of the object, adjusting the position of the optics, or adjusting the proximity of the lens array within the focal plane of the camera, all along the z axis of the optical train. In other words, objects could be made to appear to be floating in front of, at the surface of, or inside the lens array, or a combination thereof; simply by making one of these adjustments in a precise manner.

Unfortunately, some form of spherical distortion artifacts were common by virtue of the requirement of a relatively large-aperture, wide-angle, primary lens, or concave mirror. Additionally, the cameras were still only capable of imaging relatively small actual objects.

The first one step imaging solution was proposed in 1968 by A. Chutjian and R. J. Collier of Bell Labs (12). This method presented a calculated, computer-generated pseudoscopic (reversed depth) image to the lens array, which naturally re-inverted the image to be orthoscopic (correct depth). The image was formed by moving a series of progressively-changing contours of an image, in layers, on a CRT screen, or by presenting a succession of computer-written transparency film masks behind which was placed a high-intensity light source, along the optical z axis; Fig. 10. The result was a fully volumetric, computer generated image. The image was recorded through a integral lens array to a light sensitive emulsion. Not only was this the first one step method proposed, it was also the first method to create computer-generated integral imagery and the first method to propose using a CRT or transparency masks to simulate a non-existing object.

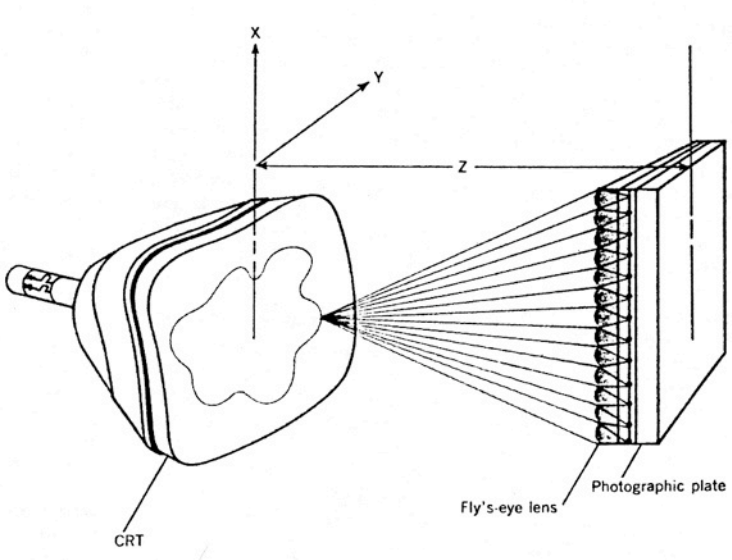

Figure 10: Illustration from Collier article in Physics Today, 1968 (11).

A similar one-step integral method was later designed by David Roberts and Ivars Villums in 1989 for Three Design Company in Wisconsin which introduced the use of color transparency masks and an objective lens camera, which Bell labs had not contemplated. The method used a dual primary lens camera, that resulted in color objects that appeared to float above or below the

lens array. In one experiment an image of a credit card was produced to appear to float several inches off the lens screen. This work was done as an extension to US Patent 4,878,735; Villums, 1989 (81).

A number of researchers advanced the process of Integral Photography in the sixties and seventies including, most prominently, Roger de Montebello (21-24) who produced hundreds of striking images using his patented Integram system (Fig. 11). He was the first to offer the technology, along with lens arrays, to the general public through his company MDH Products of Ann Arbor, Michigan. De Montebello went on to describe methods of manufacturing his lens arrays "comprising a large number of closely adjacent or contiguous lenslets, closely packed in either a square, or preferably a hexagonal array, formed of transparent moldable or castable plastic material" in 1971 (22).

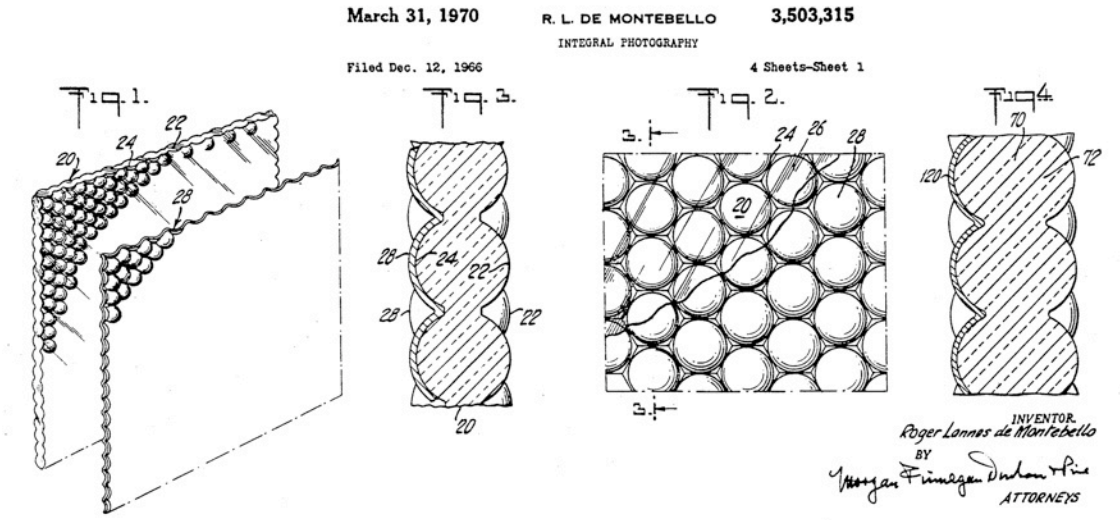

Figure 11: Integram method, U.S. Patent 3,503,315; de Montebello 1970 (21) .

Figure 12: Two views of a single 11" x 14" Integram photograph; Roger de Montebello, 1977 (23).

From 1970 through 1987 a group of researchers in the Soviet Union produced an impressive body of work that was documented in over twenty technical papers published in the Soviet Journal of Optical Technology ((1)(3)(31-38)(48-49)(70-74)). Certainly the most thorough investigation of the technology up to that time, their work still stands as some of the best reference material on the subject. The researchers included Yu. A. Dudnikov, B.K. Rozhkov, E.N. Antipova, N.K. Ignat'ev, I.M. Chaykina, N.P. Samusenko and M.D. Khukhrina.

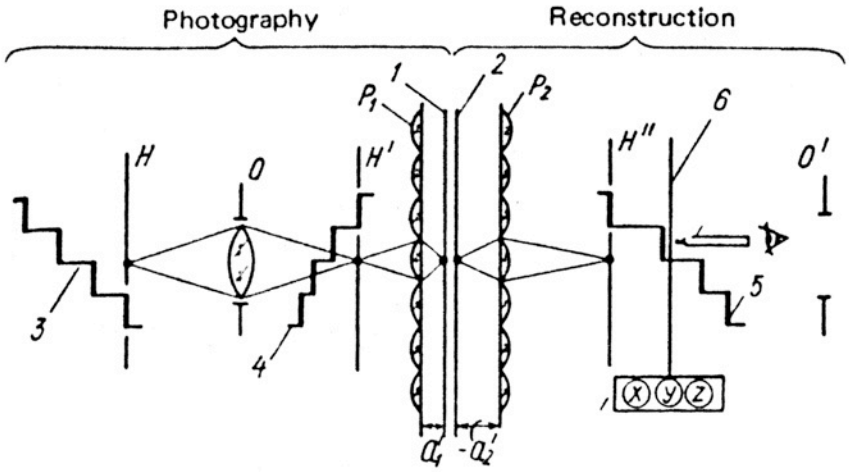

Figure 13: Scheme for photographing and reconstructing integral image: $\mathbf{P_1}$ - photographic lens array, $\mathbf{P_2}$ - projection array, ***O*** - objective lens, ***O'*** - integral image of the exit pupil of the object lens, **1** - object, **2** - image of the object beyond the objective lens, **3** - integral image of the object, **4** - ground glass plate, **5** - measurement mechanism, **6** - microscope; Rozhkov 1979 (70).

A group of researchers at the University of Sheffield UK also began investigating the process in 1988, and continue to this day. They are principally Neil Davis, Malcolm McCormick, Mike Hutley and Li Yang ((13-20)(58)(83)). They have likely done more work in the area over the last fifteen years than any other group, including developing a number of elegant pseudoscopic reversal methods including retro-reflective solutions, novel camera and lens designs, computer generated images and image reproduction methods. They now reside at the De Montfort University in Leicester under the name Imaging Technology Group.

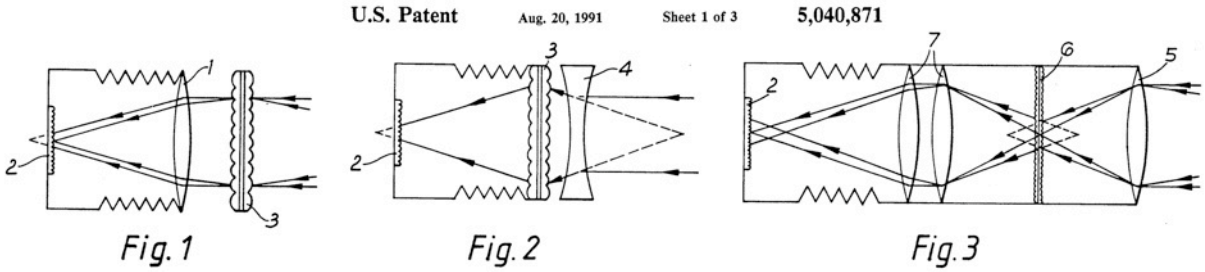

Figure 14: Pseudoscopic reversal method, U.S. Patent 5,040,871; Davis 1991 (14).

In the 1970's lens arrays were still very difficult to fabricate, and prohibitively costly, limiting the wide spread commercial potential of the method. One elegant solution was to use tiny glass

beads (typically with a diameter of 50 microns) embedded in photographic emulsions. The first to propose the concept was John Alofs of Grand Rapid Michigan in 1970 (2), closely followed in 1972 by C. B. Burckhardt and E.T. Doherty of Bell Labs in New Jersey (9). 3M continued this area of research in 1987 using the glass bead arrays to create animation effects (66) and animation effects on curved surfaces (5).

In 2001, 3M developed a three-dimensional product and method using beaded screens to form "floating" virtual objects using a high intensity laser that is scanned by a galvanometer directly to the beaded lens screen through a series of optics (40). In this approach the primary lens or lens screen is moved along the optical z axis as the image is drawn to achieve a fully volumetric image. This is recorded through the glass bead lens array to a metal-based material layer that is generally ablated, or altered thermally, to form an image. Also proposed was again the use of a mask, containing a transparency of a logo for instance, that would be made to appear to float above, within or behind the screen.

One major drawback to the use of glass beads is the focal point of a sphere actually lies well beyond its back surface, meaning the imagery produced from them was not as sharp and detailed as it could have otherwise been using a more traditional lenslet design that focused on a flat back surface (called a *plano convex* lens). Further, a simple spherical shape is not an optimal lens shape. In the early days, plano convex lenslet designs, for both integral arrays and lenticular arrays, were largely limited to spherical lens shapes. Use of such designs in imaging and image viewing can be limiting however as distortions from spherical aberration, astigmatism and coma are unavoidable. This is described by Snell's Law of refraction. Optical designers have long known the performance improvements possible by incorporating non-spherical surfaces, or aspheres, into their designs. Most lenticular designs have incorporated aspheric correction since the eighties, as have many integral lenslet designs. As the methods to master optical lens arrays improve, so does their performance by improved surface quality and precise control over shape. Today lens arrays are typically either diamond tooled into copper or nickel, or produced in photo resists by lithographic methods

Early on integral lens arrays found some interesting photographic uses that were unrelated to three dimensional imaging. The use of the arrays in creating traditional cinematic animations was suggested as early as 1932 by Eliot Keen of New York (52). His method utilized each individual lenslet to record a scene, one at a time, over a short period. These views would be played back individually in rapid sequence to show a ten or fifteen second animation sequence, all on one piece of film ordinarily used for a single exposure. The camera was used both for taking the animation and as a projector to play it back. Similar approaches were later refined by Edwin Land of Polaroid in 1960 (53), Goodbar in 1963; Fig. 15 (42), and Browning in 1966 (6). Another novel use was described by V.C. Ernest in 1935, that used lens arrays to produce lithographic halftones (39).

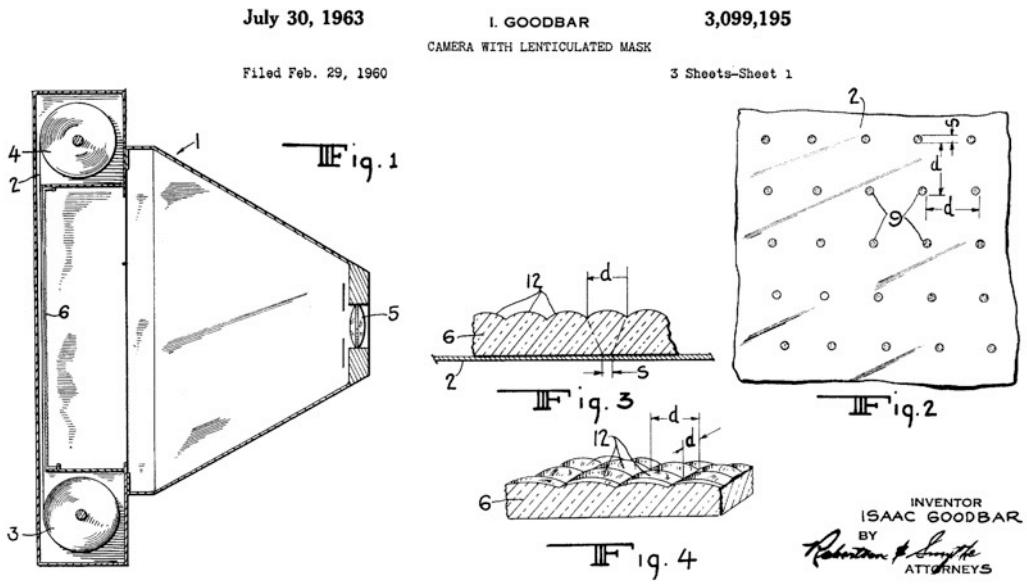

Figure 15: Camera with lenticulated mask, U.S. Patent 3,099,195; Goodbar 1963 (42).

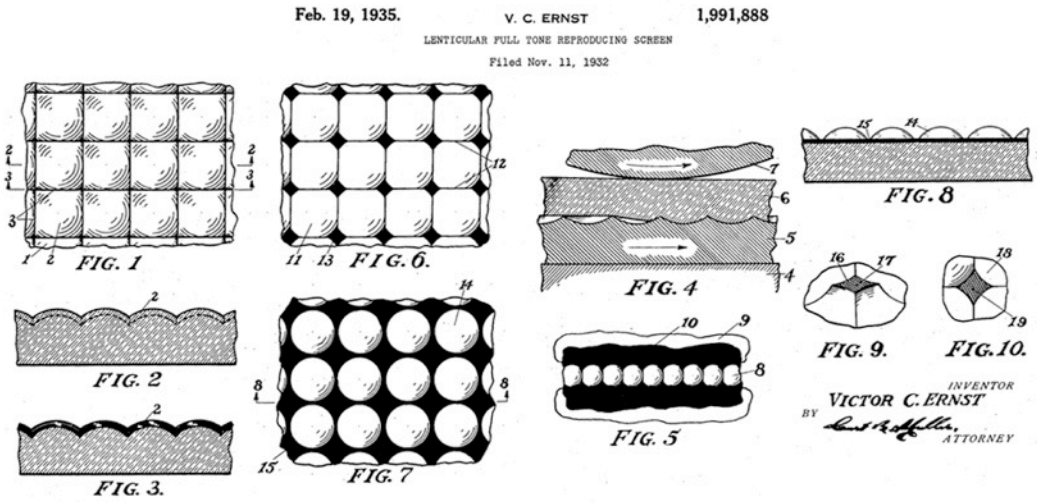

Figure 16: Full tone reproducing screen , U.S. Patent 1,991,888; Ernst 1935 (39).

The first in depth study of lithographic printing of auto-stereo imagery was described in 1936 by Carl Percy and Ernest Draper of the Perser Corporation; Fig 16. It included methods of mass producing integral imagery, either using a lens array or aperture array, what they referred to as *diclinic* imagery, and lenticular or barrier, what they called *monoclinic* imagery.

Along with describing methods to form the lens arrays in celluloid or glass, they point out the problem of objectionable moiré artifacts that resulted when using traditional halftone printing methods to reproduce the images. Two general solutions to the moiré problem were proposed. The first solution was to use halftone screening, but at non-standard screen angles. They wrote,"It is best to avoid angles whose tangents are equal, or nearly equal, to the ratio of any two integers (considering zero as an integer)". "That is, it is best to avoid the following angles: Arc tan 0/1 = 0°, Arc tan 1/1 = 45°, Arc tan 1/2 = 26-1/2°, Arc tan 1/3 = 17°, Arc tan 2/3 = 34°". They further recognized that auto-stereo imagery required higher definition than traditional printing, and thereby suggested using line screens with frequencies as high as 400 lines per inch, over four times the norm of the period.

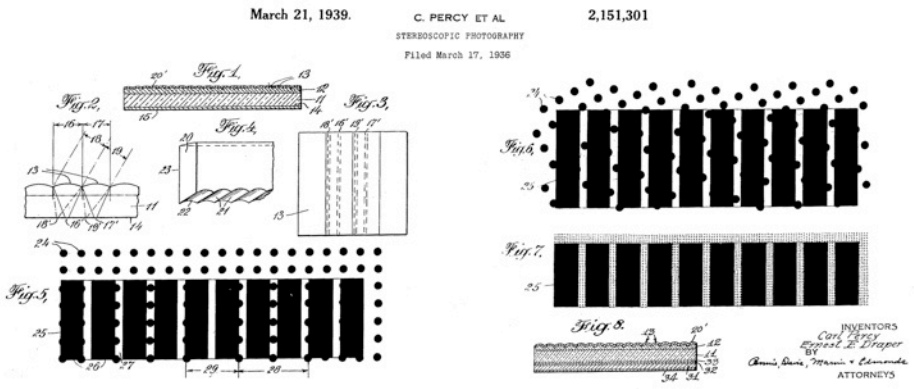

Figure 16: Percy and Draper demonstrate the advantages of rotating halftone screen angles to avoid moiré patterning, U.S. Patent 2,151,301; 1936 (69).

The second solution was to use a continuous tone printing method of the era called collotype, which was a gelatin plate process, exposed through photographic film, that did not posses a mechanical dot structure, and therefore avoided moiré effects altogether. The method had it's drawbacks however, as the gelatin plates were fragile, which resulted in very short press runs of typically only a few hundred sheets, and required very skilled and specialized pressman.

The Perser corporation of New York was apparently the first company to mass produce backlit barrier strip novelty images in the thirties called Depth-O-Graph's. Their insight at the time however was remarkable, as rotated halftone screen angles, higher frequency line screens, and continuous tone (stochastic) methods are all still essential tools in lens-array printing today.

A motorized retro-reflective animation display incorporating a fly's eye lens array was proposed in 1947 by Fred Hotchner of Los Angeles; Fig. 17. In his design, an "interlaced" patterned screen, which was printed to a retro-reflective surface, was moved precisely under a lens array to create a dynamic animation effect.

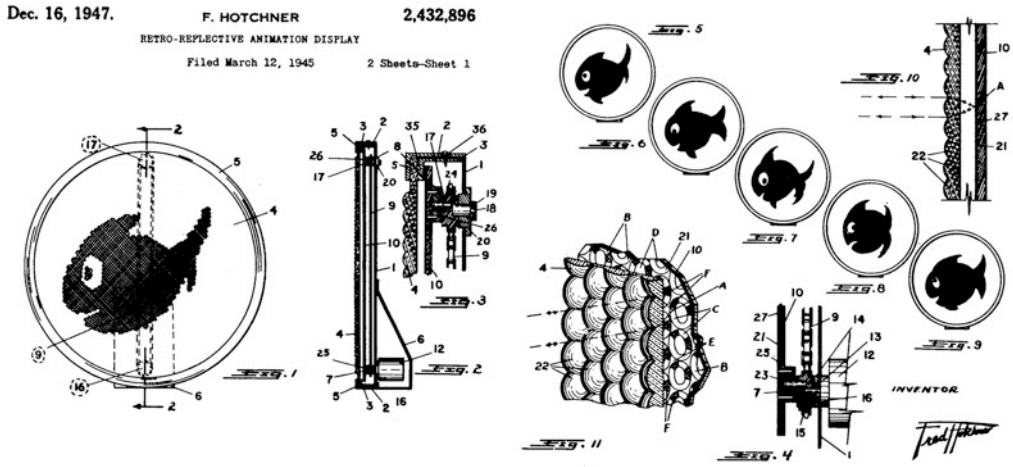

Figure 17: Retro-reflective animation display, U.S. Patent 2,432,896; Hotchner 1947 (46).

The first traditional integral animation effect method was proposed in 1958 by Juan Luis Ossoinak of Argentina who described using square-packed lens arrays or lenticular screens to produce animation flip and motion effects; Fig. 18. He suggests using the animation of "legends, mottos, photographs, cinematographic pictures or animated drawings, etc." His work was certainly synchronous with Victor Anderson's around that same period that produced flip and animated lenticular images in the millions (beginning with the famous "I Like Ike" button), but went further to consider the advantages of using an integral screen. Interestingly, Ossoinak only describes a method of arrangement of sub-images behind the lens array and the resulting animation effects, not any specific method to produce the arrangement.

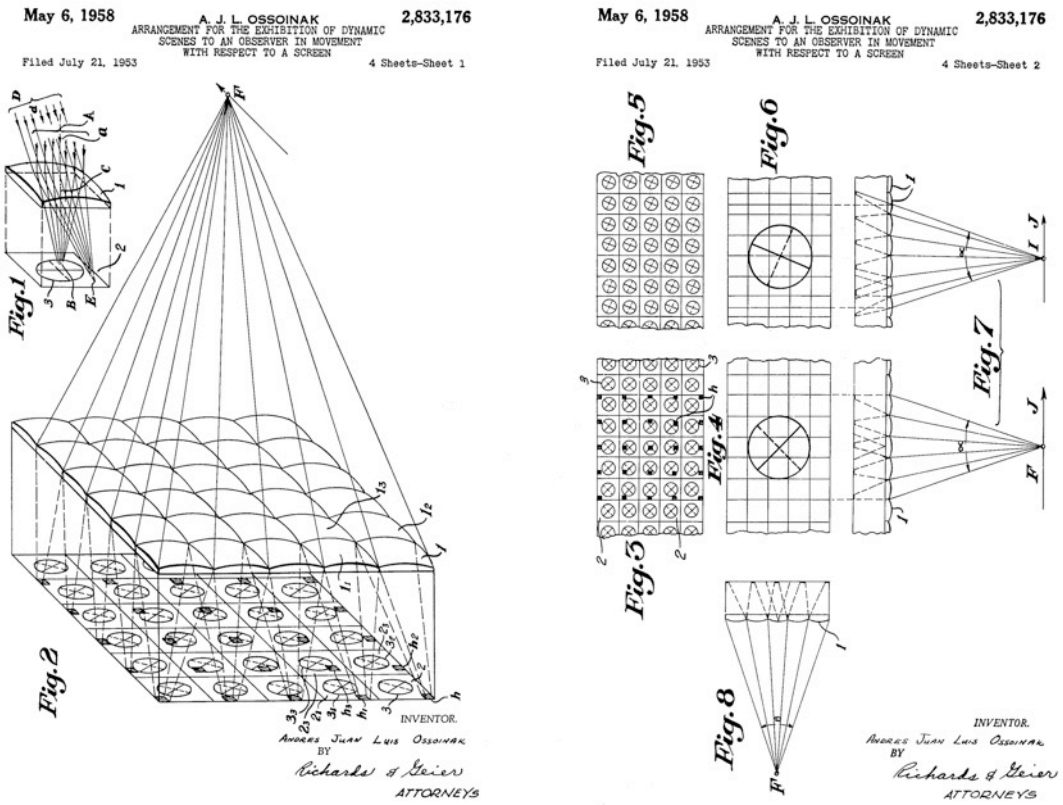

Figure 18: Integral lens array animation effects, U.S. Patent 2,833,176; Ossoinak 1958 (67).

Creating 3-D integral imagery of purely computer generated objects, by digitally calculating and interlacing image points was first demonstrated in 1978 by Yutaka Igarashi, Hiroshi Murata and Mitsuhiro of the Tokyo Institute of Technology in Japan; Fig. 19. They displayed their computer generated images on a CRT monitor, with a square packed lens array fitted to the front of the screen. The experiment used an array consisting of 53 x 53 lenses. Certainly one big advantage of computer-interlaced imagery is eliminating the need for complicated pseudoscopic-inversion methods, by simply arranging the micro images in proper orientation.

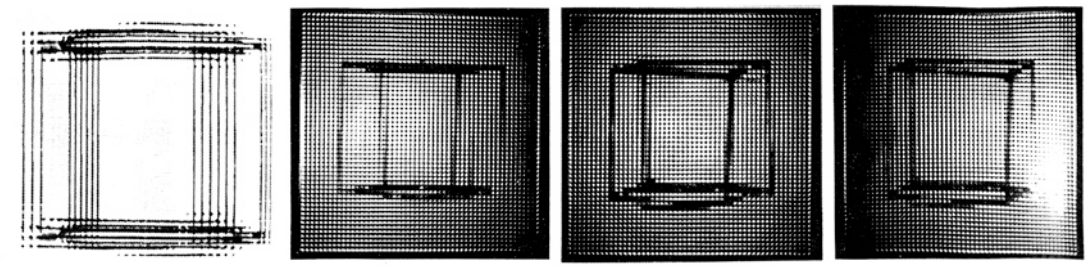

Figure 19: Digitally interlaced integral image (far left), various views of subsequent integral print (right); Japan Journal of Applied Physics, 1978 (47).

As photo-mechanical techniques were replaced by digital solutions in the late eighties and nineties, researchers naturally shifted their focus from photographic reproduction methods to digital methods. Digitally interlacing high-resolution integral imagery for output on printing devices was first proposed by Ivars Villums in 1989; Fig 20. Villums describes a method of integral imaging using diffraction-based lens arrays. Villums and David Roberts explored a variety of reproduction methods from 1988 through 1992, including x-ray imaging, photographic methods, projection methods and lithographic printing. The use of diffractive lens arrays for integral imaging continues to be explored today by Mathias Hain et al. (44).

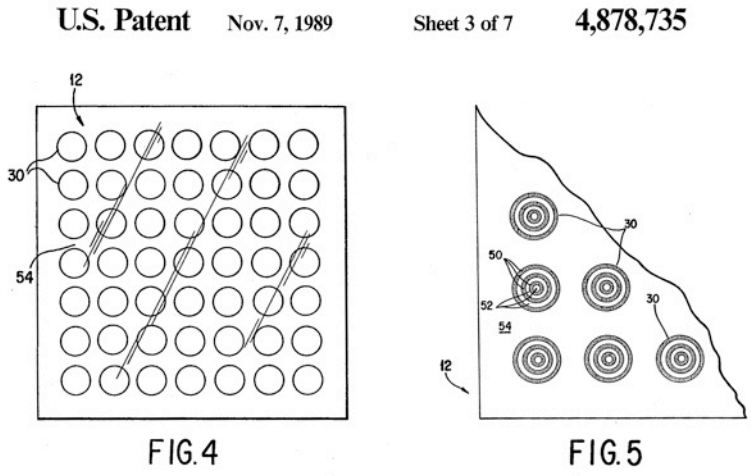

Figure 20: Diffractive integral lens array, U.S. Patent 4,878,735; Villums 1989 (81).

Following the success of mass-produced lenticular products, mass-production of integral-based products was explored by a number of companies. In 2000, Lenticular Corporation of Wisconsin began exploring the engraving of integral lens embossing extrusion cylinders via laser-ablation with the hope of mass producing the product. To support this product Satori Vision of Virginia developed FlyCom, an integral interlacing software program, Fig 21. Bringing Bonnet's groundbreaking work in the 1950's with Relief and Movement Photography (4) into the digital realm, FlyCom enabled the combination of full X/Y parallax and animation effects within digitally interlaced, integral images.

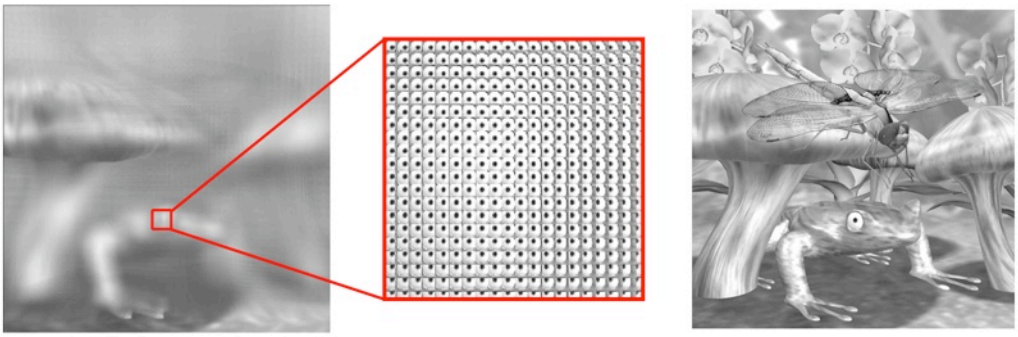

Figure 21: Digitally rendered, integral image; FlyCom, 2000. Image rendered using 100 views per lenslet (10x10); Left: interlaced integral image, Center: enlargement, Right, single de-interlaced view of 3D/Animation scene.

Since the late 1980's, Ken Conley of Microlens Technologies has been producing experimental integral-array master embossing cylinders, with numerous material runs utilizing UV resin casting at Rexham Corporation of New York, and plastic extrusion runs at Eastman Chemical in Tennessee.

Work in lithographic integral image reproduction continues with the work of Phil Gottfried of Texas in 2002 (41), and Davies and McCormick of the De Montfort University in Leicester. Dr. Daniel L. Lau of the University of Kentucky-Lexington and Trebor Smith of ThreeFlow, Inc., have also developed a number of patent-pending lenticular and integral-specific digital halftoning methods, Fig. 22.

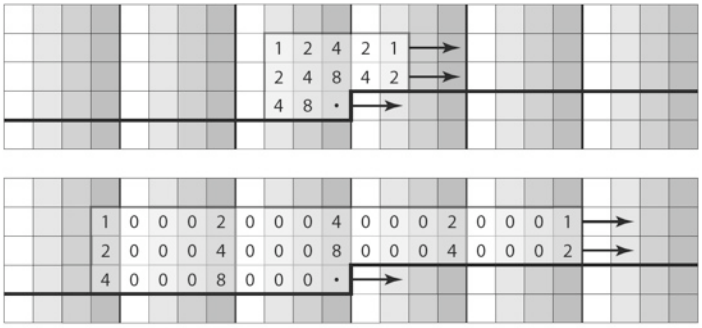

Figure 22: The (top) traditional and (bottom) lenticular Stucki error-diffusion filters for a four component, lenticular image where halftoning can now be done after the spatial multiplexing of images but with the same results as if done prior to; Dr. Daniel L. Lau, Optics Express, 2006 (54).

Integral Imaging holds great promise. While the mass production of integral lens arrays remains difficult, they will likely become widely accessible in the near future as the relevant replication technologies continue to evolve. Once available, these lenses, when coupled with readily-available digital interlacing and effects generation software, will enable lithographic integral imagery to develop as an important advertising medium.

Many thousands of experimental images have been produced throughout the last century, by a wide variety of methods, exhibiting 3-D, animation and other impressive effects. Research and commercialization of integral methods remains very active today including a wide body of work in integral television and other electronic displays. Although integral imaging has not yet achieved significant commercial success, its widespread use is inevitable.

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